USN | | | | | | | | | | |

10MAT11

# First Semester B.E. Degree Examination, January 2013
# Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

**Note: 1.** ***Answer any FIVE full questions, choosing at least two from each part.***
**2.** ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
**3.** ***Answer to objective type questions on sheets other than OMR will not be valued.***

## PART – A

1 a. Choose correct answers for the following : **(04 Marks)**

i) If $y = 3^{2x}$ then $y_n$ = ____: A) $2^{3x}(2\log 3)^n$ B) $3^{2x}(\log 3)^n$ C) $3^{2\log x}$ D) $3^{2x}(2\log 3)^n$

ii) If $y = \log(1-x)$ the $y_n$ = ____: A) $\frac{(-1)^{n-1}n!}{(1-x)n}$ B) $\frac{(-1)^{2n-1}(n-1)!}{(1+x)^n}$ C) $\frac{(-1)^{2n-1}(n-1)!}{(1-x)^n}$ D) $\frac{(-1)^{2n+1}(n-1)!}{(1-x)^{n+1}}$

iii) By Rolle's theorem the number C = _____ when $f(x) = x^2 - 4x + 8$ in [1, 3]: A) 1 B) 2 C) 3 D) 4

iv) By Maclaurins series, the expansion $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \text{--------}$ is equal to ____: A) $e^x$ B) cosx C) sinx D) x cosx

b. Find the $n^{th}$ derivative of $x^2 \sin 3x$. **(04 Marks)**

c. Show that $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$, if $0 < a < b$ and deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$. **(06 Marks)**

d. Expand $\tan^{-1}x$ in powers of x -1 upto the term containing fourth degree. **(06 Marks)**

2 a. Choose correct answers for the following : **(04 Marks)**

i) $\underset{x\to 0}{\text{Limit}}\left[\frac{\log\sin ax}{\log\sin bx}\right]$ = _______ : A) 1 B) a/b C) b/a D) ab

ii) The angle between the radius vector and the tangent of the curve $r = \sin\theta + \cos\theta$ is _____
A) $\pi/2 + \theta$ B) $\pi/4 + \theta$ C) $\pi/3 + \theta$ D) $\pi/6 + \theta$

iii) Derivative of arc length for polar curve, the value $ds/d\theta$ = _____
A) $\sqrt{r^2 + \frac{d^2r}{d\theta^2}}$ B) $\sqrt{r + \left(\frac{dr}{d\theta}\right)^2}$ C) $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ D) $\sqrt{r^2 + \left(\frac{d\theta}{dr}\right)^2}$

iv) Radius of curvature of $y = x^2$ at x = 1 is = _____ : A) $5\sqrt{5}$ B) $\frac{4\sqrt{5}}{2}$ C) $\frac{3\sqrt{5}}{2}$ D) $\frac{5\sqrt{5}}{2}$

b. Evaluate $\underset{x\to 0}{\text{Limit}}\left[\frac{a^x + b^x + c^x + d^x}{4}\right]^{1/x}$. **(04 Marks)**

c. Find the angle of intersection between the curves $r^2\sin 2\theta = 4$ and $r^2 = 16\sin 2\theta$. **(06 Marks)**

d. Find the radius of curvature at any point t of the curve $x = a(\cos t + \log\tan t/2)$, $y = a\sin t$. **(06 Marks)**

3 a. Choose correct answers for the following : **(04 Marks)**

i) If $F(u) = \sin u = \frac{x^2y^2}{x^2+y^2}$ the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ = _____ :A) cot u B) tan u C) 2 tanu D) 3 tan u

ii) Jacobian for $x = r\cos\theta$, $y = r\sin\theta$ is ____: A) r B) $1/r^2$ C) 1/r D) $r^2$

iii) The necessary condition for u = f(x, y) have maxima or minima is
A) $\partial u/\partial x \neq 0$, $\partial u/\partial y \neq 0$ B) $\partial u/\partial x = 0$, $\partial u/\partial y = 0$ C) $\partial u/\partial x > 0$, $\partial u/\partial y > 0$ D) $\partial u/\partial x < 0$, $\partial u/\partial y < 0$

iv) The percentage error in the area of the rectangle when an error of 1.0% is made in measuring the sides x and y is _____ : A) 4 B) 3 C) 2 D) 1

b. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find the Jacobian of (x, y, z) with respect to r, θ, ϕ. **(04 Marks)**

c. Find the percentage error in computing resistance r of two resistances $r_1$ and $r_2$ connected in parallel of both $r_1$ and $r_2$ are in error by 2%. **(06 Marks)**

d. Find the extreme values of the function $f(x, y) = x^3y^2(1 - x - y)$. **(06 Marks)**

4 a. Choose correct answers for the following : **(04 Marks)**

i) If $\vec{R}$ is a position vector of any point P(x, y, z) then $\nabla \cdot \vec{R}$ is ____: A) 0 B) 1 C) 2 D) 3

ii) Any motion in which the curl of the velocity vector is zero, then the vector $\vec{V}$ is said to be
A) solenoidal B) Vector C) Constant D) Irrotational

iii) If ϕ is the scalar point function then the value of curl (grad ϕ) = ____: A) > 0 B) < 0 C) 0 D) ∞

iv) In orthogonal curvilinear coordinates the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ is = ____
A) $h_1 h_2 h_3$ B) $1/h_1h_2h_3$ C) $h_1/h_2h_3$ D) $h_1h_2/h_3$

b. Show that the vector field $F=(x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find its scalar potential. **(04 Marks)**

c. Prove that $\nabla\left(\phi\vec{A}\right) = (\nabla\phi)\cdot\vec{A} + \phi\left(\nabla\cdot\vec{A}\right)$ where ϕ is a scalar field. **(06 Marks)**

d. If $\vec{F}(u,v,w)$ be the vector point function given interms of orthogonal curvilinear coordinates as $F = F_1e_1 + F_2e_2 + F_3e_3$, find curl $\vec{F}$. **(06 Marks)**

## PART – B

5 a. Choose correct answers for the following : **(04 Marks)**

i) If $I(\alpha) = \int_0^1 \left[\frac{x^\alpha - 1}{\log x}\right] dx$ then $\frac{dI(\alpha)}{d\alpha}$ = ______ : A) $4/(1+\alpha)$ B) $3/(1+\alpha)$ C) $2/(1+\alpha)$ D) $1/(1+\alpha)$

ii) The value of $\int_0^\pi \sin^4 x\, dx$ is = _____: A) $3\pi/8$ B) $3\pi/16$ C) $3\pi^2/8$ D) Zero

iii) A curve $r = a(1 + \cos\theta)$ has the length on x-axis (the initial line) ____: A) a B) 2a C) -2a D) 3a

iv) Special points on x and y-axis for the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ are ____: A) $\pm a$ B) $\pm 2a$ C) $\pm 3a$ D) $\pm 4a$

b. Differentiate under the integral sign and hence evaluate the integration $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)} dx$. **(04 Marks)**

c. Evaluate $\int_0^{2a} x^2\left(\sqrt{2ax - x^2}\right) dx$. **(06 Marks)**

d. Trace the curve $r = a(1 + \cos\theta)$ and hence find the total length. **(06 Marks)**

6 a. Choose correct answers for the following : **(04 Marks)**

i) The solution of the differential equation $dy/dx = e^{x+y}$ is _____
A) $e^x/e^y = c$ B) $e^y/e^x = c$ C) $e^x + e^{-y} = c$ D) $e^{xy} = c$

ii) If $M(x, y)dx + N(x, y)\,dy = 0$ is said to be exact then the condition is
A) $\partial M/\partial y \neq \partial N/\partial x$ B) $\partial M/\partial y = \partial N/\partial x$ C) $\partial M/\partial y > \partial N/\partial x$ D) $M = N$

iii) The integrating factor for $(x + 2y^3)\, dy/dx = y$ is I.F = ____: A) log y B) $e^y$ C) 1/y D) y + 1

iv) For $r = f(\theta)$, the replacement of $dr/d\theta$ to find the orthogonal trajectory is _____
A) $-r\frac{dr}{d\theta}$ B) $-r^2\frac{dr}{d\theta}$ C) $-r^2\frac{d\theta}{dr}$ D) $-r\frac{d\theta}{dr}$

b. Solve $(4x + 6y + 5)\, dy = (3y + 2x + 4)\, dx$. **(04 Marks)**

c. Solve $dy/dx + x \sin 2y = x^3 \cos^2 y$. **(06 Marks)**

d. Find the orthogonal trajectory of the system of confocal conics $x^2/(a^2+\lambda) + y^2/(b^2+\lambda) = 1$ where $\lambda$ is the parameter. **(06 Marks)**

7 a. Choose correct answers for the following : **(04 Marks)**

i) The system of linear equations is said to be consistent then the relation between R(A) and R(A:B) in AX = B is ___: A) R(A) > R(A:B) B) R(A) < R(A:B) C) R(A) ≠ R(A:B) D) R(A) = R(A:B)

ii) The rank of the matrix $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ is ____: A) 0 B) 1 C) 2 D) 3

iii) A square matrix is said to be symmetric matrix is ___:A) $a_{ij} = a_{ji}$ B) $a_{ij} > a_{ji}$ C) $a_{ij} < a_{ji}$ D) $a_{ij} = -a_{ji}$

iv) In Gauss elimination method the system of equations is transformed into an ____
A) Row matrix B) Column matrix C) Null matrix D) Upper triangular matrix

b. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$. **(04 Marks)**

c. Investigate the value of $\lambda$ and $\mu$, so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have i) Unique solution; ii) No solution; iii) An infinite number of solutions. **(06 Marks)**

d. Solve the system of equations by Gauss Jordon method: $2x+5y+7z = 52$, $2x+y-z = 0$, $x+y+z = 9$. **(06 Marks)**

8 a. Choose correct answers for the following : **(04 Marks)**

i) Vectors $x_1$, $x_2$, $x_3$ ----- are said to be ----- in a relation $k_1x_1 + x_2k_2 + k_3x_3 + ----- + k_rx_r$ with $k_1$, $k_2$ ----- $k_r$ are the scalars not all zero: A) Linearly independent B) Linearly dependent C) Consistent D) Inconsistent

ii) A matrix A is called orthogonal if: A) A = A′ B) A/A′ = I C) AA′ = I D) A′/A = I

iii) Eigen values of the matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$ are ____: A) 1, 3 B) 1, 4 C) 1, 5 D) 1, 6

iv) A homogeneous polynomial of second degree in n variables $x_1$, $x_2$ ----- is called a _____
A) Canonical form B) Linear form C) Exponential form D) Quadratic form

b. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular, write down the inverse transformation. **(04 Marks)**

c. Find the Eigen values and the corresponding Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. **(06 Marks)**

d. Reduce the quadratic form $x^2 + 5y^2 + z^2 + 2yz + 6xz + 2xy$ to the canonical form and specify the matrix of transformation. **(06 Marks)**